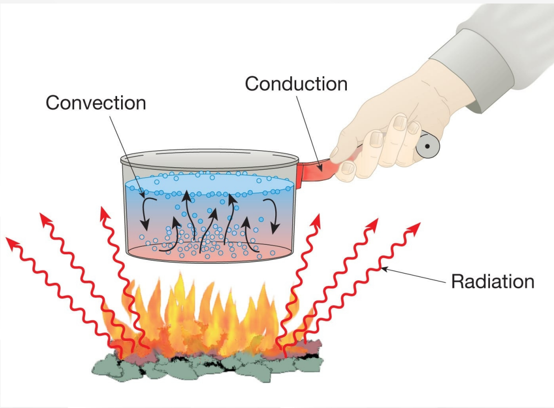
Basics Of Heat Transfer



To calculate the operational power required to maintain the desired temperature throughout the steady-state operation of the system, one must first calculate the three modes of heat loss: **conduction, convection, and radiation.**

Conduction heat loss is the loss of heat through a solid medium. In classical examples, this is depicted with a temperature difference on either side of a wall. In the case of etched-foil heaters, this heat loss can be seen in the insulation layer of the heater and through the heat sink.

Convection heat loss is the loss of heat through a fluid medium, which can be either a gas or a liquid. This mode of heat loss comes in two forms: external and internal flow. External convection can be seen in any scenario where the fluid is not enclosed but rather freely flowing around an object. Internal convection is the fixed flow of a fluid through an enclosed space, such as a pipe. For the purposes of this white paper, we will be strictly analyzing external flow over an isothermal plate.

Radiation heat loss is the loss of heat through infrared energy. In nature, radiation heat transfer can be seen from the sun. The sun undergoes radiation heat loss in order to heat our planet. In the case of etched-foil heaters, radiation can be seen in most applications, as it is the loss of heat to the surroundings.

Assumptions

The following assumptions are required for these calculations. Without these assumptions, the formulas used throughout the calculations will need to be changed accordingly.

1. Steady-State Processes
2. Constant Properties
3. Uniform Heat Distribution
4. No Energy Generation

Disclaimer

All of the values calculated using this white paper can only be viewed as estimations. It is up to you, as the reader, to make the final determination for your application.

Calculating Initial Required Power

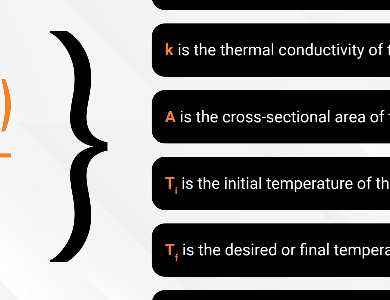
If you can assume that there is no heat loss in your application, then the simplest form of calculating your required power (in watts) for your thermal system is with the following formula.

$$P_i = \frac{mc_p(T_f - T_i)}{3.4121t}$$

- P_i is the power required for the heater in watts [W]
- **m** is the mass of the material in pounds [lbs.]
- C_p is the specific heat of the heat sink material [BTU/lbs/°F] (Table 1)
- T_i is the initial temperature of the material in Fahrenheit [°F]
- T_f is the desired temperature of the material in Fahrenheit[°F]
- **t** is the desired warm-up time for the heater in hours [hrs]
- **3.4121** is a conversion factor for 1 [BTU/hr] to 1 [W]

However, in most applications, you will not be able to simply ignore the heat loss. You may be able to disregard one or two modes, but it is more than likely that at least one mode will need to be accounted for. The following three formulas describe how to calculate each mode of heat loss.

Conduction

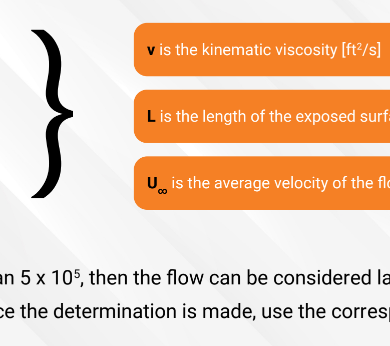


In order to calculate the conduction heat loss from your application, you will first need to collect some information regarding your setup. You will need the thermal conductivity of the heat sink, k, which can be determined from the tables in the appendix, the thickness of the insulation, and the cross-sectional area of the heat sink where the heat will be applied. You will also need to know the initial temperature of the heater and the desired final temperature of the heat sink. When you have all this information, you can find the power or heat loss of the application using the following formula:

$$P_{cond} = \frac{kA\ (T_f - T_i)}{3.4121L}$$

- P_{cond} is the power loss due to conduction [W]
- **k** is the thermal conductivity of the heat sink [BTU/(hr-ft¬-°F)] (Table 2)
- **A** is the cross-sectional area of the heat sink [ft^2]
- T_i is the initial temperature of the heater [°F]
- T_f is the desired or final temperature of the heat sink [°F]
- **L** is the thickness of the insulation [ft]

Convection



To properly calculate the heat loss due to convection, a determination must be made about the convection coefficient, h. This value is determined through a series of dimensionless numbers known as the Reynolds Number (Re), Prandtl Number (Pr), and Nusselt Number (Nu). In order to use these numbers, the final temperature is required to look up a few values in the tables: the Prandtl Number, kinematic viscosity, and thermal conductivity (Table 2). Once these values are found, the Reynolds Number is calculated to determine if the flow is turbulent or laminar.

$$Re = \frac{U_\infty * L}{v}$$

- **v** is the kinematic viscosity [ft^2/s]
- **L** is the length of the exposed surface parallel to the flow [ft]
- U_∞ is the average velocity of the flow [ft/s]

If the Reynolds Number is less than 5 x 10^5, then the flow can be considered laminar, but if it is greater than 5 x 10^5, then the flow is turbulent. Once the determination is made, use the corresponding formula to calculate the Nusselt Number.

$$Nu_{laminar} = 0.453Re^{1/2}\,Pr^{1/3}$$
$$Nu_{Turbulent} = 0.0308Re^{4/5}\,Pr^{1/3}$$

- **Re** is the Reynolds Number
- **Pr** is the Prandtl Number

Once the Nusselt Number is found, the convection coefficient can be calculated using the formula below:

$$h = \frac{Nu * k}{L}$$

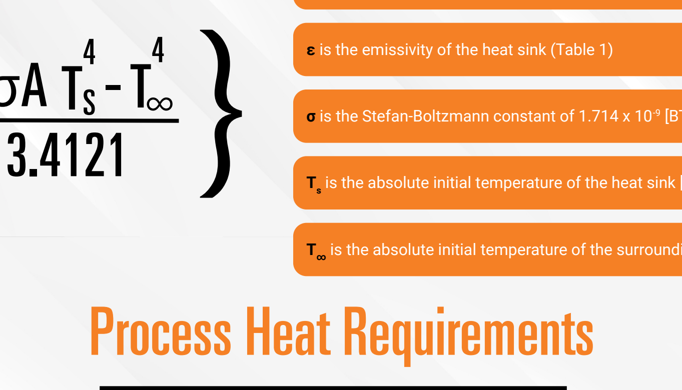
- **Nu** is the Nusselt Number
- **k** is the thermal conductivity [BTU/(hr-ft¬-°F)]
- **L** is the length of the exposed surface parallel to the flow [ft]

Finally, after the convection coefficient is calculated, the power or heat loss from convection can be calculated using the formula below:

$$P_{conv} = \frac{hA\,[T_s - T_\infty]}{3.4121}$$

- P_{cond} is the power loss due to conduction [W]
- **h** is the convection coefficient [Btu/(hr-ft^2-°F)]
- **A** is the exposed area [ft^2]
- T_s is the surface temperature [°F]
- T_∞ is the fluid temperature [°F]

Radiation



A few more pieces of information are required to calculate the radiation heat lost, the first being the emissivity of the heat sink. This value is represented with the Greek letter ε. For this calculation, it is important to change your units of Fahrenheit to Rankine, or your calculations will not be correct. To convert to Rankine, add 460 to any temperature recorded in Fahrenheit—this is called the absolute temperature. Once you have collected these last bits of information, you are ready to calculate the radiation heat loss using the following formula:

$$P_{rad} = \frac{\varepsilon\sigma A\ T_S^4 - T_\infty^4}{3.4121}$$

- P_{rad} is the power loss due to radiation [W]
- **ε** is the emissivity of the heat sink (Table 1)
- **σ** is the Stefan-Boltzmann constant of 1.714 x 10^{-9} [BTU/(hr*ft^2*°R^4)]
- T_s is the absolute initial temperature of the heat sink [°R]
- T_∞ is the absolute initial temperature of the surroundings [°R]

Process Heat Requirements

This is the final calculation to complete before calculating your operating heat requirements and can be skipped if the sole purpose of the heater is to maintain temperature. If your heater is used for warming, melting, or vaporizing other materials, then this heat loss needs to be considered when calculating your operating power requirements. Use the following formula to find your process power requirements:

$$P_{proc} = \frac{mC_p(T_f - T_i) + U}{3.4121t}$$

- **m** is the mass of the material [lbs.]
- C_p is the specific heat of the material [BTU/lbs/°F]
- T_f is the final temperature of the material [°F]
- T_i is the initial temperature of the material [°F]
- **U** is the latent heat of fusion/vaporization [BTU/lbs] (only if material changes state; otherwise, set to 0)
- **t** is the process time [hrs]

Determining Operating Power Requirements

Once you have calculated all the required heat loss modes, you can estimate the required operating power by summing all of the previously calculated values. If your system does not require one or more forms of heat loss, those terms can be omitted from the formula. The most general form is as follows:

$$P_{op} = P_{cond} + P_{conv} + P_{rad} + P_{proc}$$

Caution

All values calculated using this white paper are strictly estimates. To determine your actual required power, you must operate the heater on your equipment under actual environmental conditions.

Calculating Warm-up Power

Only calculating your power requirements for the steady-state operation is not sufficient. To determine your required power, you must compare the steady-state operating power to the warm-up power. The greater of the two values will be the estimated power required. To calculate your warm-up power, you will need three values: the initial required power, the phase change during warm-up, and the losses during warm-up. The initial required power was covered at the start of this paper. The phase change and losses during warm-up will be covered in this section.

If your system contains material that will pass through a phase change, this energy will need to be calculated. Otherwise, this part can be set to 0 in the final equation. The phase change energy calculation for vaporization and fusion is as follows:

$$E_{phase\ change} = U * m$$

- $E_{Phase\ Change}$ is the energy required for the material to change phase[W]
- **U** is the latent heat of fusion/vaporization [BTU/lb.]
- **m** is the mass of the material undergoing the phase change [lbs.]

The losses during warm-up can be estimated to be 65% of the losses during steady-state operation. Taking the power requirements calculated above for each necessary loss, the warm-up losses can be calculated as follows:

$$Pwarm\ loss = 0.65\ (P_{cond} + P_{conv} + P_{rad})$$

- $P_{Warm\ loss}$ is the power required to account for the warm-up losses [W]
- P_{cond} is the power loss due to conduction [W]
- P_{conv} is the power loss due to convection [W]
- P_{rad} is the power loss due to radiation [W]

Finally, the warm-up power can be calculated using the following formula:

$$P_{warm\text{-}up} = P_i + E_{phase\ change} + P_{warm\ loss}$$

Determining Power Requirements

In order to determine the estimated minimum power required for your application, you need to compare your calculated values for the warm-up power and the operational power. Whichever value is higher will be your minimum total power required.

Appendix

Table 1: Thermophysical Properties of Selected Solids at 540 Rankine

Material	Density ρ [lbs/ft^3]	Specific Heat CP[BTU/(lbs-°F)]	Thermal Conductivity k [BTU/(hr-ft°F)]	Emissivity ε (room temp.)
Aluminium, Pure	169	0.216	128	0.07
Bronze(90 -10)	550	0.1	2.50	0.1
Bronze(70 -30)	525	0.1	70	0.55
AISI 1010	489	0.104	3.08	
Copper, pure	558	0.1	228	0.03
Gold	1205	0.003	172	0.03
Inconel	530	0.11	8.67	
Iron, pure	491	0.11	3.86	
Lead	708	0.031	20.08	0.08
Nickel, Pure	551.68	0.106	4.37	0.05
Platinum, Pure	1339.01	0.031	40	0.08
Silver	655	0.056	242	0.03
Stainless steel, AISI 302	502.86	0.115	0.73	
Stainless steel, AISI 347	498	0.115	0.68	
Tin	456	0.056	39	0.07
Titanium	281	0.126	11.05	
Zinc	445	0.095	15.67	

Source: BERGMAN, THEODORE L. (2020). FUNDAMENTALS OF HEAT AND MASS TRANSFER. WILEY, S.l.

Table 2: Thermophysical Properties of Air at Atmospheric Pressure

Temperature T[R]	Density ρ [lbs/ft^3]	Specific Heat C_p[BTU/(lbs-°F)]	Kinetic Viscosity v*(10^6) [ft^2/s]	Thermal Conductivity k*(10^3) [BTU/(hr-ft-°F)]	Prandtl Number
180	0.222	0.2465	21.53	5.40	0.780
360	0.109	0.2405	81.70	10.46	0.726
540	0.0725	0.2405	171.04	15.20	0.707
720	0.0544	0.2422	284.27	19.53	0.699
900	0.0435	0.2460	417.53	23.52	0.698
1080	0.0362	0.2510	567.15	27.10	0.703

Source: BERGMAN, THEODORE L. (2020). FUNDAMENTALS OF HEAT AND MASS TRANSFER. WILEY, S.l.